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DYNAMIC SYMMETRY: A CRITICISM

I

THOUGH published under university auspices and enlisting for its preparation the interest and seemingly the sympathy of the classical specialists of two of our great American museums, Mr. Hambidge's treatise on dynamic symmetry[1] has thus far elicited very little archaeological comment. Yet his theory, if true, is of fundamental importance for Greek esthetic theory and for our understanding of the relations between mathematics and artistic practice in antiquity. In certain modern circles, among lay theorists and practical designers, Mr. Hambidge is said to have succeeded in gaining a considerable following for his methods; but it is not as a working formula for artistic production today, so much as a brilliant and novel explanation of the structural formulae of sixth and fifth century Greek vases that Mr. Hambidge's treatise commands the attention of classical archaeologists, who owe its author their gratitude for turning his labor and their attention so searchingly to the fundamentals and minutiae of the formal structure of many of the most beautiful shapes of ancient pottery.

Those who had not the fortune of initiation through personal instruction had for long heard distantly of Mr. Hambidge's process of "Dynamic Symmetry" as of some great thaumaturgy through which the theory of ancient design had been fundamentally affected. When at last Mr. Hambidge published his book on structural design, based on accurate and very detailed measurements of Greek vases in the Boston, New York, and New Haven museums, his expectant public among classical archaeologists felt, with disappointment, that the subject had not been made as accessible as had been hoped. Very remarkable properties were apparently inherent in ancient vases; but there was no simple and direct statement of what dynamic symmetry was, or how it was to be detected, or what artistic properties it imparted. The shadow of a geometric mysticism seemed to obscure the issues. Before

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[1] Jay Hambidge, *Dynamic Symmetry: the Greek Vase.* Yale University Press. 1920.

American Journal of Archaeology, Second Series. Journal of the Archaeological Institute of America, Vol. XXV (1921), No. 1.

one can appreciate, one must understand; but the understanding of dynamic symmetry had been left extremely difficult.

From Mr. Hambidge's treatise it is apparent that dynamic symmetry has to do with the relation of surface areas in a design. "Static symmetry" (which is treated as its antithesis) depends upon simple commensurability of lengths, of linear measurements. But dynamic symmetry, apparently, is not mere commensurability of area. An ellipse and a circle having twice the area of the ellipse are apparently not an instance of dynamic symmetry. Mr. Hambidge confines his instances to rectangles. The computation of areas of curvilinear outline would be distressingly difficult. Accordingly, in the case of Greek vases, it is *not* the area of the vases which is computed. The actual area of the vase surfaces, I may say, is nowhere computed, and is a matter of indifference. For the curvilinear area of the vase a simple rectangle is substituted. This is the containing rectangle, of which the sides are parallel to the vertical axis and the base-line of the vase. It is, as it were, the smallest rectangular frame into which the whole vase will fit. To this rectangle the analysis for dynamic symmetry is applied. Not its size, but its shape, is important. If this rectangle can be split up into rectangles of similar and related shapes, and if these smaller rectangles can be used to determine recognizable elements of the vase, the occurrence of dynamic symmetry is held to be established.

In a sense, the first condition can always be fulfilled geometrically, since within any rectangle an infinite number of rectangles of similar and related shapes can be constructed. But dynamic symmetry apparently requires not merely that similar rectangles shall be discoverable, but that the whole rectangle may be completely subdivided into squares and rectangles similar to the original rectangle or of closely related shape. This process of subdivision is the chief geometrical element in dynamic analysis. Such analysis of a rectangle is usually accomplished (1) by division into halves, thirds, etc., or (2) by laying off the shorter side on the longer side (so as to form a square) and then treating the remainder of the original rectangle as a new rectangle subject to similar analysis. The completed process will thus show a disintegration of the original rectangle into squares and rectangles. These rectangles will be similar to the original rectangle only under certain conditions. Rectangles fulfilling these conditions are the only ones used for dynamic symmetry. Mathematically it is perfectly easy to formulate these conditions and discover what rectangles will satisfy them.[1]

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[1] As this is nowhere clearly performed in Mr. Hambidge's book, a brief notice may be of service:

(1) When the short side is laid off on the long side of a rectangle so as to form a square, the requirement that the remainder of the rectangle shall be similar to the original rectangle may be stated thus,

$$x-1=\frac{1}{x}$$

where x is the long side, and the short side is 1. This equation will be satisfied if $x=\frac{\sqrt{5}+1}{2}$ (*i.e.* very nearly 1.618). A rectangle with its sides in the ratio of 1.618 to 1 will, therefore, satisfy the condition. This particular 1.618 shape is called by Mr. Hambidge the "Whirling Square Rectangle" and is, next to the square, the most frequent form in dynamic analysis.

(2) Since $\frac{\sqrt{x}}{x}=\frac{1}{\sqrt{x}}$, it follows that rectangles of which the sides are to each other as the square root of an integer is to 1, will have a special property of subdivision into shapes similar to the whole. Substituting the value 2 for x, it follows that when a "Root-Two Rectangle" (*i.e.* one of which the longer side is to the shorter as $\sqrt{2}$ is to 1) is cut into 2 parts (*sc.* at the mid point of the longer side) each part will be a "Root-Two Rectangle"; similarly (substituting 3 for x) when a "Root-Three Rectangle" (*i.e.* with sides as $\sqrt{3}$ to 1) is cut into 3 parts, each part will be a "Root-Three Rectangle" (and so on for higher values of x). Because of this property of subdivision, the "Root-Rectangles" are peculiarly suitable for "dynamic" analysis.

(3) The "Root-Five Rectangle" (*i.e.* with sides as $\sqrt{5}$ to 1) is related to the "Whirling-Square Rectangle." Since $\frac{\sqrt{5}-1}{2}$ is the reciprocal of $\frac{\sqrt{5}+1}{2}$ (by equation 1) it may be simply stated that a "Root-Five Rectangle" is made up of a square plus two "Whirling-Square Rectangles," because $1+2\left(\frac{\sqrt{5}-1}{2}\right)=\sqrt{5}$.

The geometric analysis of "Dynamic Symmetry" is therefore based on these three equations:

(1) $x-1=\frac{1}{x}$ when $x=\frac{\sqrt{5}+1}{2}$ ("Whirling-Square Rectangle").

(2) $\frac{\sqrt{x}}{x}=\frac{1}{\sqrt{x}}$ ("Root Rectangles").

(3) $\sqrt{5}=\frac{\sqrt{5}-1}{2}+\frac{\sqrt{5}-1}{2}+1$ (analysis of "Root-Five Rectangle" into square and "Whirling-Square Rectangles").

The remarkable subdivisibility of the rectangles used in "Dynamic Symmetry" is thus due to certain simple inherent mathematical properties of the particular rectangles selected. It is not due to the potter nor to the construction of the vase.

Practically, there are only five forms of rectangles used, *viz.*, those with the following ratios between their sides:

(1) 1: 1 (Square).

(2) $\sqrt{2}$: 1 ("Root Two Rectangle").

(3) $\sqrt{3}$: 1 ("Root Three Rectangle").

(4) $\sqrt{5}$: 1 ("Root Five Rectangle").

(5) 1.618: 1 ("Whirling Square Rectangle").

This does not imply that all Greek vases were made within only five bounding shapes, but rather that all Greek vases are contained in rectangles which can be cut up into parts every one of which can be classed as one of these five shapes. Even so qualified, this assertion is sufficiently startling; but Mr. Hambidge shows in his book how true it is.

Now it is perfectly obvious that no merit or ulterior motive or artistic subtlety can be ascribed to a vase just because its containing rectangle can be so subdivided. God, "the eternal geometer," and not the potter, must here have the credit. But if the simplest and most obvious divisions of the containing rectangle frame the various distinct elements of the vase, so that there is a coincidence between the geometrical and the ceramic construction, then it is hard to believe that this occurs by chance; we credit the potter with the intention.

When we consider this coincidence of rectangles with the elements of the vase, we discover that, after all, it is not really as much a question of *areas* as we might have supposed. The geometry is all in rectangular areas; but the coincidence of these areas with the vase is a matter largely of points on lines. Thus a certain area will establish the width of the lip; but it is not properly the *area* of the lip which is so determined, it is its linear horizontal extension. Actually, it is mainly the linear measurements along horizontal and vertical axes which are determined by this geometry of rectangular areas.

A glimpse of Mr. Hambidge at work will make this clear. In Figure 1[1] the whole rectangle is divided vertically into three rectangles standing side by side. The middle one of the three (CQD) touches the ends of the narrowest part of the bowl. On the vase there is no question of area: the least width of the bowl is one-third the extreme width of the vase,—a purely linear (or "static") measurement. Next, either of the end rectangles is subdivided horizontally into three (according to a perfectly legitimate formula), and the narrow central rectangle so formed is again divided into three (again perfectly legitimately) so as to produce two squares and between them a rectangle similar to the great rectangle of which it occupies the centre. In a very spectacular way, a vertical dropped from the centre of one of these squares just strikes the extreme point of projection of the base of the scyphus (S) and a prolongation of one side of the other square just strikes the extreme point of projection of the lip of the scyphus (H). My only aim at present is to make it clear that the *areas* of these squares find no echo in the areas of lip or base, but that the geometry of rectangular areas merely fixes the end *points* of these parts of the base. It determines *linear* measurements.

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[1] *Op. cit.* p. 105, fig. 1.

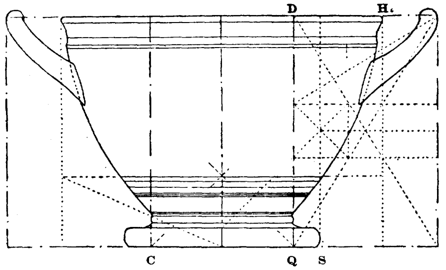

Figure 1.—Scyphus Analysed by Dynamic Symmetry.

Such a scyphus, then, though composed with "dynamic symmetry," displays to the eye no commensurable areas. We cannot say simply that dynamic symmetry is the coördination of areas in the same sense that "static symmetry" (ordinary linear proportion) is the coördination of lengths. This clearly would not be true.

Rather it would seem that dynamic symmetry is a method for establishing linear measurements according to ratios which would not otherwise be directly intelligible. In the scyphus illustrated above, if the width of the narrowest part of the bowl be called 1, the width of the base would be 1.382 and the width of the lip would be 2.236, while the height of the scyphus would be 1.618 and its extreme width from handle to handle would be 3. Only in this last figure would the uninitiated detect any coherence or rational intention.

In the cantharus[1] shown in Figure 2, in Mr. Hambidge's opinion "one of the finest of Greek cups," the containing area is divided into squares and into rectangles of the same shape as those in our previous figure. The width of the stem, the height of the

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[1] *Op. cit.* p. 68, fig. 8.

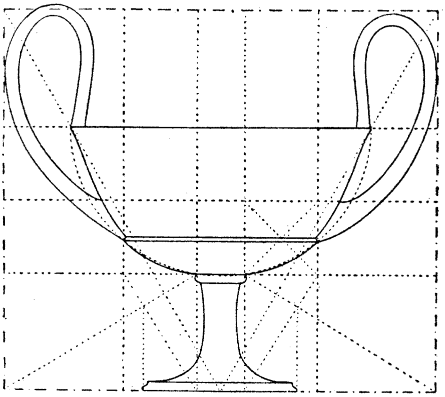

Figure 2.—Cantharus Analysed by Dynamic Symmetry.

bowl, the length of the moulded base-line for the painted figures, are all fixed by these squares and rectangles; but no discrete element of the cup coincides with any of these areas. On the cup (Fig. 3)[1] there are no squares or rectangles to be seen. The eye cannot help dwelling on the beautifully running contours; but these contours run through and across the rectangular geometry, and only in the instance of the base-line and the top line of the bowl is there any agreement between outline of vase and outline of rectangles. Even this agreement is highly theoretical, since there are no straight lines in the cantharus at all. The straight edge of the lip is really the curve of a full circle and looks curved from almost every point of view. From the one point of view in which it appears as a straight line (when the cantharus is held vertically and with the rim at the level of the eye) the base-line will look curved; while if the base-line looks straight, the rim will look curved. The geometrical analysis of the figure is based on an orthogonal projection of the vase upon a single vertical plane. Like an architect's elevation of a façade, it is true, but unnatural. If it is thought that this is a quibble over minutiae, the hastiest measurement performed on the photograph of this vase will convince one that the consideration

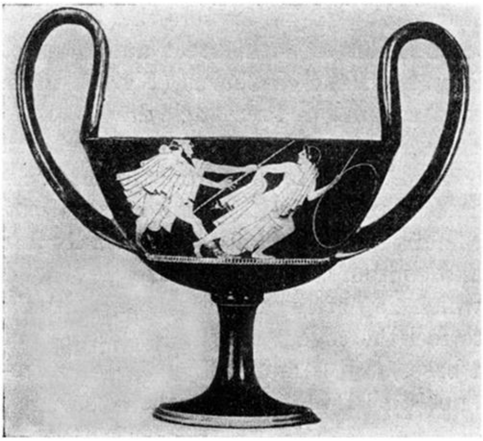

Figure 3.—Red-Figured Cantharus.

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[1] *Op. cit.*, pl. facing p. 68.

is important, since the vase in the photograph will not measure up to the ratios in the diagram, even approximately. The actual vase has to be measured with caliper and ruler for true diameters and heights and widths, before the ratios will work out.

All this serves to indicate that a vase composed in dynamic symmetry will not reveal commensurable areas to the spectator nor even provide him with data from which such area relations can be readily inferred.

In our contemplation of a vase we can scarcely be expected to correlate rectangular air-spaces with others which overrun the edges of the vase, since a fundamental requirement of esthetic contemplation is the isolation of the artistic object from its accidental surroundings. The boundary lines of a piece of sculpture are much more truly barriers the more we concentrate our attention upon artistic qualities. It is a perverse requirement that we should treat the contours of a beautiful vase as practically nonexistent in order to overlap them with imaginary rectangles. We can do it on paper; but in the presence of an actual and material vase, we are conscious that the region where surrounding air ends and vase begins is as crucial as the imaginary plane in a playhouse where audience ceases and stage commences.

Moreover, many of the dimensions are determined by much more elaborate geometrical constructions than the mere subdivision of the containing rectangle into subsidiary ones. Diagonals, intersecting diagonals, and verticals or horizontals drawn from such intersections, play a very important part. "It seems to have been recognized early that diagonals were the most important lines in the determination of both direct and indirect proportions."[1] In the diagram of the cantharus (Fig. 2) a line from either upper corner of the containing rectangle drawn to the mid-point of the base cuts across the ends of the rim of the bowl and thus fixes its length. A vertical dropped from the intersection of two diagonals of related rectangles fixes the width of the base. Could any Greek, holding up this cantharus, with its fine play of rounded shapes and contours, mentally make these constructions of rectangles which lie partly on the vase and partly in the empty air, so as to feel the cogency and the rightness of these lengths of rim and base, as determined by such intersections? Clearly that is not the intention. Rather we must imagine that all this geometrical manipulation is a guide not for the spectator, but for the potter; and perhaps because of it, some quality of rightness or beauty or relevance accrues to the vase, so that its results are somehow felt, though the geometric means are not understood. This seems to be Mr. Hambidge's contention. To the employment of dynamic symmetry he attributes "the quality of inevitableness." He admits (p. 103) that the "key-plan" of a "well-trained designer who understands his symmetry . . . . . will be unintelligible to any inferior in symmetry knowledge to himself"; and suggests (*ibid.*) that "dynamic symmetry produces in a design the correlation of part to whole observable in either animal or vegetable growth. It is a satisfying harmony of functioning parts which suggests a thing alive or a thing which has the possibility of life." "Beauty, perhaps, may be a matter of functional coördination."

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[1] *Op. cit.* p. 50.

Dynamic symmetry depends, then, on the area subdivisions of the containing rectangle, but is not a method for fixing areas. It determines certain crucial points in the design, but apparently leaves the potter free to choose his outlines between these points and so make his areas what he will. The dynamic rectangles will be unaffected by his choice of contours, since these rectangles are part empty air and part orthogonal projection of vase upon a plane surface. We had always imagined that the curving outlines of ancient Greek vases were their supreme artistic quality and the source of our delight in them. For Mr. Hambidge these outlines are seemingly irrelevant. As far as any statement in his book goes, I can see no ground for holding that *any* vase constructed on points coincident with the intersections which occur in the geometry of Figure 2 is a whit less beautiful than this cantharus. Yet many designs which would satisfy the geometric analysis would be extremely ugly in outline.[1]

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[1] On pp. 126 f. Mr. Hambidge hints at a "method of relating curves to the straight line and area proportion." He gives as an instance a *deinos* ("a static example") and shows how the curve runs tangent to the diagonals of certain areas. But as these areas are all composed of uniform squares, the instance is exceptional. Mr. Hambidge says further: "Hardly a vase, among the hundreds so far examined, fails to disclose this method of relating curve to angle, area, and line. The constructions necessary to show this have been kept out purposely in other examples to avoid confusion." We can only regret that no diagrams illustrative of this method of relating curves were included. Rectangles can certainly be drawn so that their diagonals shall lie tangent to the

Dynamic symmetry must accordingly be at most only a contributory influence and not a sole and sufficient cause for beauty in a design. As always, there are here no mathematical formulae which can guarantee to a craftsman inevitably his artistic success. But if that is the case, what (in less vague and general phrases than those which we have quoted) does the use of dynamic symmetry really contribute to a design?

To answer this question we must watch the process of vase analysis a little more closely.

II

We have seen that the shape (not the size) of the containing rectangle is the starting-point for further analysis. It is clear that (quite apart from size) the shape of a rectangle depends upon the *relative* lengths of the longer and shorter sides. The ratio of length to width is a sort of index of the shape. To determine this index we may measure the length of the sides (by any scale) and divide one measurement by the other. The resulting numerical value will be the same for all rectangles of similar shape, no matter what their size or what scale of measurement we have used. "The first step is the approximate determination of the containing rectangle. This is done arithmetically from direct measurement."[1]

Since very few Greek vases have a width or height more than three times their other dimension, the index of their shape will nearly always be a number between 0 and 3. Arithmetically, the index is carried out by Mr. Hambidge to three places of decimals to insure accurate distinction. One would expect to find all imaginable values between 0 and 3. In Mr. Hambidge's book one learns that certain combinations of figures are favorites and tend to recur frequently. Thus 1.236, 1.382, and 1.854 occur in many instances. Why 1.236 each time, and not 1.234 or 1.239? It is only fair to say that 1.236 is only an approxima-

contour curve of any vase; but it is not immediately apparent that the contour can be constructed by means of diagonals to the rectangles actually used in the dynamic analysis of the figures of Mr. Hambidge's book. The consideration seems so important for the whole theory of dynamic symmetry that we may confidently expect further and more explicit details from Mr. Hambidge and his collaborators.

[1] *Op. cit.* p. 102.

tion: 1.236 yields dynamic results, whereas 1.234 and 1.239 do not. Therefore in the "approximate determination of the containing rectangle," any index falling close to 1.236 may be classed as an instance of that particular number. I do not know just how great a margin of error is allowable. Mr. Hambidge tells us that he "has found that the small errors found in Greek pottery, except in few cases, are practically negligible," and that "it is really better to make the small corrections necessary to true up an example" (p. 68); and again that "the percentage of error is much smaller in the bronzes than in the pottery" (p. 76), so that in the clay vases, from which most of the examples are derived, some deviation from the exact three-place decimal of the correct index is always to be expected.

To advance from this stage of the analysis, the student must know the kind and character of the rectangle revealed by this numerical index. A peculiar family of rectangles lurks in such indices as .618, 1.236, 1.382, 1.618, 1.809. Another family includes .5858, .7071, 1.3535, 1.4142. Clearly this is a matter of familiarization with certain effective numbers, and requires a certain amount of practice or study of Mr. Hambidge's geometrical manipulations.

Granted that the index is one of these effective numbers, the next step is the disintegration of the rectangle into component squares and rectangles which (though of varying sizes) will all have the same index, or an index belonging to the same family. Thus, in Figure 1, all the subdivisions are either squares or rectangles, of which the index is .618 (*i.e.* one-third of the index of the containing rectangle, to which it is, therefore, closely related). In Figure 2, all the subdivisions are squares, or .618, or .382 rectangles, which are all related to one another and to 1.118, the index of the containing rectangle.

The object of this manipulation is, as we have seen, to fix points which shall coincide with important end-points of the elements of the vase. Thus in Figure 2, the end-points of the rim, the base, the ring at the top of the stem, and the moulded groundline for the painted figures, are so determined.

There is no rule or law which demands that corresponding points or measurements in vases of a similar shape or class shall be fixed by the same specific subdivisions of the rectangle. Any point may be fixed in any way, provided that the rectangles are always constructed so as to be similar or related, and that no intersections shall be used unless they be produced by diagonals of such rectangles. If this condition is fulfilled, the intersection of any two available diagonals may be used to fix any point on the same horizontal or vertical axis. Sometimes it is the extreme projection of a base moulding or lip moulding, sometimes it is the inner edge of these mouldings, sometimes it is the imaginary projection of the contour of the bowl across base or lip, which is so determined. In consequence, it is impossible to give a clear statement of the artistic advantage which such a haphazard procedure might be expected to impart to a vase. We must be content with the vague phrases which Mr. Hambidge gives us and which we have quoted.

In defence of Mr. Hambidge's method it should be urged that, after all, it is noteworthy that (1) Greek vases are, by actual measurement, contained within just these rectangles which possess such conspicuous properties of subdivision rather than in other rectangles without these properties, and (2) there are so many coincidences between the actual vase and a geometry with such restricted rules of play. These two arguments are vital, and deserve attention and discussion.

Is it not significant that, when so many vases have been so accurately measured, there should be such a persistent conspiracy in favor of these peculiar rectangles? How is it that, allowing for a margin of error, the indices always lie so close to one of these effective numbers?

If the method of arithmetical analysis is closely studied, it will transpire that the crowd of these effective numbers is rather larger than one would suppose. Any simple multiple or submultiple of an effective number is an effective number; any effective number added to unity or to twice unity or to half unity, or subtracted from unity, is an effective number. In determining the index, if the length of the shorter side is divided by the length of the longer side, a fraction between 1 and 0 must necessarily result; while if the division is performed the other way round (longer side by shorter side) a number greater than 1 (and probably less than 3) will result. Consequently every index above unity has a corresponding index below unity (its reciprocal) and the ranks of effective numbers are thus doubled. The following is an incomplete list of effective numbers between 1 and 3, occurring in the vase analyses in *Dynamic Symmetry*. To each one of these there is a corresponding reciprocal lying between 1 and 0:

1.0225
1.0356
1.045
1.118
1.1382
1.146
1.1708
1.191
1.2071
1.236
1.2764
1.2929
1.309
1.3455
1.3535
1.382
1.4142
1.4472
1.472
1.528
1.5858
1.618
1.691
1.7071
1.7236
1.764
1.7888
1.809
1.854
1.8944
1.9045
2.000
2.045
2.118
2.1213
2.1382
2.236
2.309
2.3535
2.382
2.4142
2.4714
2.472
2.528
2.618
2.7071
2.764
2.854
2.8944
2.944

If necessity arose, other combinations could be made and interpolated in the series; but these fifty are all that were actually employed for the analysis of nearly one hundred different vases, all of which display more or less clearly the presence of dynamic symmetry in their design.

If we examine the above table, we shall find that there are no intervals or gaps as great as .1, and that nearly one half the intervals are less than .03 and one fourth are less than .02. These intervals represent very small differences in the linear measurements of an ancient vase of ordinary size. If a hydria about 10 inches high yielded an index of 1.125 on first measurement, this could be classed as an instance of 1.1382 with a deviation of only one twentieth of an inch in the measurement of its height and width; or it could be considered to be an instance of 1.118 with a deviation of barely half as much. As we can scarcely hold the potters accountable to the twentieth part of an inch, it is abundantly clear that there is no difficulty in classifying any vase under one of the effective numbers in the table. The most awkward situation which could arise would be an index of 2.1871 (which falls midway in the largest gap in the table); yet a lecythus $8\frac{3}{4}$ inches high, which yielded this index, would only differ from a 2.236 or a 2.1382 lecythus by one sixteenth of an inch in its height and width; and this, be it remembered, is the most unfavorable instance that can readily be imagined.

Yet, if the identification of the containing rectangles of actual Greek vases with the rectangular shapes of which the indices appear in the above table is to a considerable extent facile and arbitrary, how does it come that the "dynamic" subdivisions of these rectangular shapes coincide with so many salient points of the vases? If the potter did not intend this particular rectangle, why does the vase agree so well with the geometry of dynamic symmetry based on this rectangle?

It is only just to point out that part of this dynamic geometry is merely an elaborate method for cutting lines into halves, thirds, and quarters. A few extreme instances of this somewhat misleading practice may here be considered.

In Figure 4[1] the ratio of total height to total width is 1.472. The containing rectangle is divided by constructing a .618 rectangle at top and bottom (CD and AB in the figure). By means of intersecting diagonals, the lower of these .618 rectangles has other .618 rectangles cut out of it (viz., JK, LP) and by these the width of the base and the foot of the bowl is fixed. A moment's reflection will show that the particular shape of these rectangles is irrelevant and that what has taken place is a dichotomy of the total width of the vase into halves and quarters. The width of the base is one half, the width of the foot of the bowl is one fourth the total width.

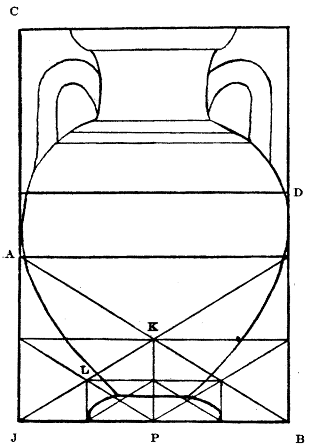

FIGURE 4.—SUBDIVISION OF AMPHORA.

In Figure 5[2] the ratio of maximum width to height is 1.809. On AD, which is one half the height, a .618 rectangle is constructed and divided into a square (DE) and a smaller rectangle (EB) which will also be .618 in shape. The intersection of the diagonals of the square "fixes the width of the bowl." The inner vertical side of the square determines the width of the base. "The points GH show that the meander band at the top of the picture is related to the foot."

Since DE is a square on half the height, this geometric construction merely goes to show that the maximum width of the scyphus is equivalent

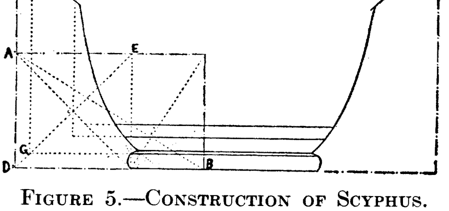

FIGURE 5.—CONSTRUCTION OF SCYPHUS.

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[1] *Op. cit.* p. 84, fig. 11.

[2] *Op. cit.* p. 109, fig. 10.

to width of base plus height of vase, and the width of the bowl is equivalent to width of base plus half the height of vase,—which looks like a convenient potter's formula, but need not be concerned with areas or dynamic symmetry.

As for G and H, since they lie on lines bisecting the corner angles of the rectangle, there is here only an elaborate geometric periphrasis for the observation that the base ring and the top meander-band are of the same height.

It may be readily imagined that such manipulation may become much more involved and that a simple result may be reached by geometric ritual so elaborate that a perfectly honest self-deception on the part of the analyzing draughtsman may ensue. In Mr. Hambidge's book there are enough instances of such geometrical periphrasis to warrant a charge of deliberate obscurantism, were it not apparent that these instances have arisen out of uncritical enthusiasm for geometric analysis.

Yet, when such bisections and trisections of lines by unnecessarily complicated "root-rectangles" have been omitted, there remains a very considerable apparatus of more relevant analysis by proportional subdivision into similar rectangles (which is the earmark of dynamic symmetry). Unless the potters employed similar geometric methods, how is it possible to account for the really extraordinary number of instances where the chief elements of the vase coincide with the divisions of this analysis?

It has already been pointed out that there is no normal and standard system of analysis. Within certain limits and according to certain rules, there is very great latitude. Any method of drawing similar rectangles and constructing diagonals is permissible. What is applicable in one design may be irrelevant in another. There is no explicit or tacit reason why such and such a combination of diagonals should fix such and such a point on the vase. If it does, that is enough: there has been dynamic symmetry.

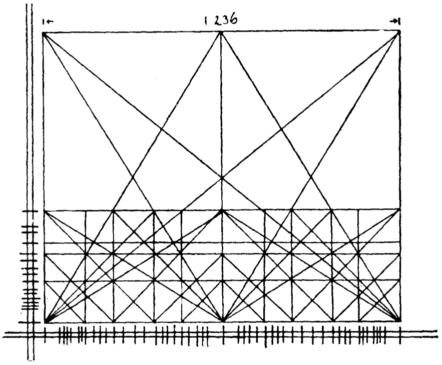

FIGURE 6.—SUBDIVISION OF RECTANGLE BY DYNAMIC SYMMETRY.

In Figure 6, a 1.236 (over-all) rectangle has been subdivided into two .618 rectangles. I have indicated the most important constructions which are admissible in the dynamic analysis of this shape, and have projected the consequent intersections horizontally and vertically upon a base line and a vertical axis. If any important element of a 1.236 vase (such as the extremity of foot, throat, lip, or the junction of foot and bowl or the level of any decorative band) agrees with any of these subdivisions, the presence of dynamic symmetry is thereby established. Other intermediate positions can be interpolated; but these are the major divisions to which, if possible, the vase analysis should be confined.

III

Shall we say that, with such a system, any vase ancient or modern could be forced into the framework of dynamic symmetry? It is tempting to answer in the affirmative; but I think that the careful student will decide that such an answer is not justified. Even if we were to eliminate 90 per cent of the analyses in Mr. Hambidge's book as mere adroit manipulation, combined with a mystifying conversion of very simple linear ratios into a guise of "root-rectangles," we should be left with an irreducible minimum of still unexplained coincidences. This minimum, however, is not nearly so great as the casual reader would be led to imagine. Many of the irrational ratios and "root-rectangles" are wholly gratuitous assumptions, and their yield of mysterious inner ratios depends upon the very unabstruse geometrical principle that similar rectangles have similar properties and that root-rectangles persistently yield other root-rectangles when they are properly subdivided.

The following is a crucial instance. On the reader's judgment of the issue here involved will hang his whole faith in, or distrust for, dynamic symmetry.

The lecythus[1] in Figure 7 is analyzed by Mr. Hambidge as follows: "The vase shape is two squares, AB and BC in the drawing. AD, the height of the bowl, is a root-two rectangle. The area CD is composed of the square DS and the root-two rectangle SN. A side of a square, ES, produced from E to J, determines the root-two rectangle JS and fixes the juncture of the neck with the body. A diagonal to the whole cuts a side of a square at G to fix the proportion of the lip. It also intersects the end of a root-two rectangle at L to determine the width of the foot at its juncture with the bowl. The line VI is the centre of the root-two rectangle AD. This is the line on which the painted figures stand. O is the intersection of a diagonal of the whole with the diagonal to the two squares AP. The point U is the intersection of the diagonal to two squares with the diagonal to the root-two rectangle NS. The points H and W are fixed by a line from C to I."[2]

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[1] The vase-outline is only roughly sketched in. The comparisons should be made mathematically.

The geometrically-minded will readily perceive that the various measurements are nearly all connected with $\sqrt{2}$ ($=1.4142$). If the potter were not conversant with $\sqrt{2}$ and rectangles of which the sides were in this ratio, how did these measurements ever get into his work?

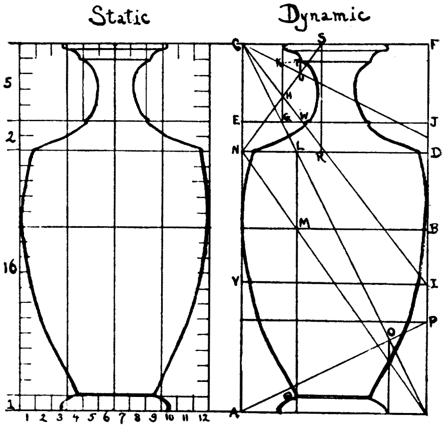

Figure 7.—Lecythus Analysed by Static and Dynamic Symmetry.

Let us suppose that the potter, knowing nothing of such rectangles, armed himself with a wand, stick, or rule, on which he had marked off 16 equal parts, as the Greeks divided their foot. He would then proceed to shape the bowl of his lecythus, making it as high as his measure, as wide as 12 of its 16 parts at its greatest width, but only as wide as 5 parts at bottom. To this bowl he would add a base one part high, a throat 2 parts high, contracting to a width of 4 parts, and set on this a neck (with a moulded lip) 5 parts high, the lip being 7 parts wide. This is all pure "static symmetry," done with a measured rule, without related areas, or $\sqrt{2}$, or root-rectangles. Yet, practically speaking, *this lecythus would be indistinguishable from that constructed dynamically* (cf. Fig. 7), and if it were drawn on paper and subjected to the same analysis of squares and diagonals, all the geometry would be the same. To be sure, there would be no $\sqrt{2}$ rectangles, and the points fixed by the geometry would be shifted a little; but the *max-*

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[2] *Op. cit.* pp. 125 f.

imum displacement would equal only .0025 of the width of the vase,—which would be a *maximum* shift of $\frac{1}{100}$ of an inch for eight inches of height. The following table will show how undetectably minute are these differences between Mr. Hambidge's dynamic vase and the suggested static one:

	Dynamic	*Static*	*Difference*
Total height of vase	2.	2.	.0000
Extreme width of vase	1.	1.	.0000
Height of bowl with base	1.4142	1.4167	.0025
Height of throat	.1716	.1666	.0050
Height of neck with lip	.4142	.4167	.0025
Width of lip	.5858	.5833	.0025
Width of foot of bowl	.4142	.4167	.0025
Width of base	.6 ?	.6 ?	?

In the above lecythus there is, therefore, very fair pretext for saying that the root-rectangles are accidental intrusions and that the ancient potter need not have had any understanding of "dynamic" symmetry.[1]

In this connection it should be noted that many of the most frequent and important "effective numbers" happen to fall very close to certain simple "static" ratios:

2.236 ($\sqrt{5}$) is scarcely distinguishable from............ 2.25 =9:4
2.00 ($\sqrt{4}$) is identical with......................... 2. =8:4=2:1
1.732 ($\sqrt{3}$) is scarcely distinguishable from............ 1.75 =7:4
1.618 (the redoubtable "whirling square" ratio) and
.618 (its reciprocal) closely approximate.............. 1.6 =8:5
.625=5:8
1.309 (a frequent ratio of the .618 family) agrees very
nearly with................................ 1.33 =4:3

Here more than anywhere else lies the key to Mr. Hambidge's ingenious magic.

It is particularly to be emphasized that a ratio approximating 5:8 has in all ages been a recurring favorite in artistic composition and artistic design. It is the famous "divine section," or "*Phi* proportion" about which so many more or less scientifically reputable studies and monographs have been written. Some-where in the neighborhood of that ratio, man has an inveterate tendency to localize his sense for beauty of proportions. For the old potter, working with a simple rule, that ratio was a natural one to employ. Continued bisection of his rule would give him 8 parts or 16 parts with which to lay out and measure. It was only to be expected that he should often avail himself of that harmonious division into a little more and a little less than half which $\frac{5}{8}$ or $\frac{10}{16}$ would give him. Wherever he used this ratio, the dynamic analyst will be able to discover "whirling squares," since $\frac{5}{8}$ is a remarkably close approximation to the division into extreme and mean proportion from which the "whirling square" rectangle derives its peculiar properties of subdivision.

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[1] So, in Figure 2, by the draughtsman's own geometry the bowl must be almost precisely twice as wide as it is high, the stem (without the little moulded base) $\frac{3}{4}$ the height of the bowl and its spread at the bottom equal to the height of the bowl. The whole cantharus can be constructed "statically" on a measure divided into 8 parts.

For example, a common potter's formula for the scyphus would seem to have been "lower diameter plus projection of bowl equals total height." In practice, the potter, having set a measure for his height, divided this into two parts, and used the smaller part for the lower radius and the larger part for the upper radius of the scyphus (cf. Fig 5). If (as he frequently did) he divided his original measure into $\frac{5}{8}$ and $\frac{3}{8}$, the scyphus area will inevitably analyze into "whirling square" rectangles.[1] Did the potter choose his formula in order to make areas which, in orthogonal projection on a plane, would reveal such .618 rectangles lying partly on and partly off the vase; or are these rectangles merely a geometric periphrasis for a few simple linear proportions which helped the potter to a satisfactory scyphus shape? Is the assumption of dynamic symmetry with its attendant paper-made geometry really indispensable? Must we hold that the sixth and fifth century potters—often slave-born humble artisans—knew all this geometry, and, since the constructions cannot be done mentally nor yet on the potter's table, used up precious parchment to draw these rectangles and diagonals? Or are we to assume that they derived their measurements, correct to a fairly small fraction of an inch, from contemplating figures drawn Archimedean-wise with a pointed stick on the ground, or with charcoal on a slab of wood? Even the humblest sixth century potter could divide his rule into eighths and sixteenths; but it is hard to convince ourselves that he ever performed the analyses with which Mr. Hambidge credits him, or that he could have inherited traditional shapes based on such analyses.

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[1] In the scyphus of Figure 1 the potter used $\frac{5}{8}$ of the height for the lower diameter and $\frac{3}{8}$ for the projection. Most of the elaborate geometry of the analysis follows automatically from this formula alone.

To sum up,—when we notice (1) the multiplicity of indices for the containing rectangles, (2) the elaborately various and seemingly arbitrary combinations of sub-rectangles and diagonals by which the chief points of the vase are established, (3) the complete irrelevance of these rectangles to the actual areas of the vase, and especially to the contour-curves which are so largely the animating life of an ancient vase, and (4) the frequent minute divergence between this intricate analysis and the simple ratios of the linear scale,—we must allow that Mr. Hambidge's discovery of a far-reaching and long-forgotten Graeco-Egyptian lore of dynamic symmetry is still very much *sub judice*. As it stands, the evidence is ingenious, but ambiguous. *A priori*, the probabilities are all against its being true.

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